[This question paper contains 6 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 768 **B**

Unique Paper Code : 42351201

Name of the Paper : Calculus and Geometry, CBCS (LOCF)

Name of the Course : **B.Sc. (Programme) Mathematical Sciences/ Physical Sciences**

Semester : II

Duration : 3 hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt any **two** parts from each question.
4. **All** questions are compulsory.
5. Marks are indicated.

P.T.O.

1. (a) Sketch the graph of the function $f(x) = 4x^3 - 9x^4$. (6.5)

(b) Find the critical points, inflection points and asymptotes (if any) for the function $f(x) = \frac{x^2+3}{x^2-4}$. Determine the region where the function increases or decreases, its concavity and sketch the graph. (6.5)

(c) Identify the symmetries of the curve $r^2 = 9\cos 2\theta$ and then sketch the curve. (6.5)

(d) Define parametric equations and parametric curves. Trace the curve described by

$x = \sin \pi t,\ y = \cos 2\pi t$ for $0 \le t \le 0.5$. (6.5)

2. (a) Evaluate the following limits using L'Hospital's Rule

$\lim_{x\to b} \frac{x^b - b^x}{x^x - b^b}$ and $\lim_{x\to\infty}\left[x - \ln\left(x^5 - 10^{15}\right)\right]$. (6)

(b) Sketch the graph of $y = (x-8)^{1/3}$. (6)

(c) Use cylindrical shells to find the volume of the solid that is generated when the region R bounded by the curve $y = x^2$, the x-axis and for $0 \le x \le 3$ is revolved about the line $y = -2$. (6)

(d) Find the volume of the solid formed when the region between the graphs of $y = 1 + x^2$ and $y = 3 - x$ is revolved about the x-axis. (6)

3. (a) Find the volume of the solid generated when the region enclosed by $y = 2\sqrt{x}$, $x = 1$, $x = 4$ and the x-axis is revolved about the y-axis. (6.5)

(b) Find the length of the cardioid :

$r = a(1 + \cos\theta)$. (6.5)

(c) Find the area of the surface generated obtained by revolving the parametric curve defined by $x = t^2$, $y = t^3$, $0 \le t \le 1$ about the y-axis. (6.5)

(d) Find the arc length of the curve defined by

$x = \frac{1}{3}t^3,\ y = \frac{1}{2}t^2,\ 0 \le t \le 1$. (6.5)

4. (a) Obtain reduction formula for evaluating the integrals of the form $\int \sin^m x \cos^n x\, dx$, m and n are positive integers. Evaluate $\int_0^{\pi/3} \sin^6 3x \cos^3 3x\, dx$. (6)

(b) Derive the reduction formula

$$\int \sec^n x dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx$$

n is a positive integer > 2. Evaluate the integral $\int \sec^4 x dx$. (6)

(c) For m, n distinct nonnegative integers, prove that

$$\int_0^{2\pi} \sin mx \cos nx\, dx = 0 .$$ (6)

(d) Show that

$$\int_0^{\pi/2} \sin^n x\, dx = \frac{n-1}{n} \int_0^{\pi/2} \sin^{n-2} x\, dx \quad (n \geq 2)$$

and $\int_0^{\pi/2} \sin^n x\, dx = \frac{\pi\, 1.3.5...(n-1)}{2.2.4.6...n}$ for even values of n. (6)

5. (a) Find equation of the parabola with axis parallel to the y-axis, vertex (5, –3) and passes through (10,2). (6.5)

(b) Describe the graph of the equation

$9x^2 + 4y^2 - 18x + 24y + 9 = 0.$ (6.5)

(c) Sketch the graph of hyperbola

$\frac{(y+4)^2}{3} - \frac{(x-2)^2}{5} = 1$ and label its vertices, foci and asymptotes. (6.5)

(d) Rotate the coordinate axes to remove the xy - term from the equation, name the conic

$x^2 - xy + y^2 - 2 = 0$ and sketch its graph. (6.5)

6. (a) Sketch the graph and show direction of increasing t for

$r(t) = 2\cos t\, \hat{i} + 2\sin t\, \hat{j} + t\hat{k}$. (6)

(b) Find where the tangent line to the curve

$$\vec{r} = e^{-2t}\hat{i} + \cos t\, \hat{j} + 3\sin t\, \hat{k}$$

at the point (1,1,0) intersects the yz-plane. (6)

P.T.O.

7. (a) (i) If $\vec{F} = (3x^2y - z)\hat{i} + (xz^3 + y^4)\hat{j} - (2x^3z^2)\hat{k}$, find $\nabla(\nabla.F)$ at the point (2,−1,0).

(ii) Prove $curl(\nabla Ø) = 0$ where $Ø = Ø(x, y, z)$. (6)

(d) Verify that for the radius vector $r = x\hat{i} + y\hat{j} + z\hat{k}$

$\nabla(1/\|r\|) = -r/\|r\|^3$ and $div\, r = 3$. (6)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 756 B

Unique Paper Code : 32351202

Name of the Paper : Differential Equation

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Use of non-programmable scientific calculator is allowed.

1. Attempt any **three** parts. Each part is of **5** marks.

(a) Solve the differential equation

$$(x^3 + y^3)dx - (x^2y + xy^2)dy = 0.$$

(b) Solve the initial value problem

$$(1 + ye^{xy})dx + (2y + xe^{xy})dy = 0, \; y(0) = 1.$$

(c) Find the general solution of the differential equation

$$xy'' + y' = 4x.$$

(d) Solve the differential equation

$$(x^3y^2 + xy)dx = dy.$$

(e) Find an integrating factor of the form $x^p y^q$ and solve the differential equation

$$(8x^2y^3 - 2y^4)dx + (5x^3y^2 - 8xy^3)dy = 0.$$

2. Attempt any **two** parts. Each part is of 6 marks.

(a) Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei has undergone disintegration in a period of 100 years.

(i) What percent of the original radioactive nuclei will remain after 1000 years?

(ii) In how many years will only one-fourth of the original number remain?

(b) A certain city had a population of 25,000 in 1960 and a population of 30,000 in 1970. Assume that its population will continue to grow exponentially at a constant rate. What population can its city planner expect in the year 2000?

(c) An arrow is shot straight upward from the ground with an initial velocity of 160 ft/s. It experiences both the deceleration of gravity and deceleration $v^2/800$ due to air resistance. How high in the air does it go?

(d) The following differential equation describes the level of pollution in the lake

$$\frac{dC}{dt} = \frac{F}{V} C_{in} - C$$

where V is the volume F is the flow (in and out), C is the concentration of pollution at time t and C_{in} is the concentration of pollution entering the lake. Let = 28×10^6 m^3, F = 4×10^6 m^3/month. If only freshwater enters the lake.

(i) How long would it take for the lake with pollution concentration 10^7 parts/m^3 to draw below the safety threshold 4×10^6 parts/m^3?

(ii) How long will it take to reduce the pollution level to 5% of its current level?

P.T.O.

3. Attempt any **two** parts. Each part is of 6 marks.

(a) Write down the word equations along with compartment diagrams that describe the movement of the drugs between the three compartments in the body, the GI tract, the bloodstream and the urinary tract, when a patient takes a single pill. Here, the urinary tract is only an absorbing compartment. From the word equations, develop the differential equation system which describes this process, defining all variables and parameters as required.

(b) Solve the logistic differential equation with the initial condition $X(0) = x_0$.

(c) A population, initially consisting of 1000 mice, has a per capita birth rate of 8 mice per month (per mouse) and a per capita death rate of 2 mice per month (per mouse). Also, 20 mouse traps are set each week and they are always filled.

(i) Write down a word equation describing the rate of change in the number of mice and hence write down a differential equation for the population.

(ii) Find the population of mice after 6 months.

(d) Consider the harvesting model

$$\frac{dX}{dt} = rX\left(1 - \frac{X}{K}\right) - h.$$

(i) Find the two non-zero equilibrium populations.

(ii) If the harvesting rate h is greater than some critical value h_c, the non-zero equilibrium values do not exist and the population tends to extinction. What is this critical value h_c?

(iii) If the harvesting rate $h < h_c$, the population may still extinct if the initial population x_0 is below some critical level X_c. What is this critical initial value X_c?

4. Attempt any **two** parts. Each part is of **6** marks.

(a) Find the general solution of the differential equation

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} - 10x \frac{dy}{dx} - 8y = 0.$$

(b) Show that $y = 1/x$ is a solution of $y' + y^2 = 0$, but that if $c \neq 0$ and $c \neq 1$, then $y = c/x$ is not a solution.

(c) Use the Wronskian to prove that the functions

$$f(x) = e^x,\ g(x) = e^{2x},\ k(x) = e^{3x};$$

are linearly independent on the real line.

(d) Use method of undetermined coefficient to find particular solution of differential

$$y''' + y' = 2 - \text{Sin } x.$$

5. Attempt any **two** parts. Each part is of **6** marks.

(a) Find the general solution of the differential equation

$$y^{(4)} - 8y'' + 16y = 0$$

(b) Solve the initial value problem

$$2y^{(3)} - 3y'' - 2y' = 0;\ y(0) = 1,\ y'(0) = -1,\ y''(0) = 3.$$

(c) Find the general solution of the Euler's equation

$$x^3y''' - 3x^2y'' + xy' = 0.$$

(d) Use the method of variation of parameters to find the solution of the differential equation

$$y'' + 3y' + 2y = 4e^x.$$

6. Attempt any **two** parts. Each part is of **6** marks.

(a) By making appropriate assumptions develop a model with two differential equations describing

predator-prey interaction with DDT spray effect. Check the model in the limiting case of

(i) Prey with no predator.

(ii) Predator with no prey.

(b) In a long-range battle, neither army can see the other, out fires into a given area. A simple mathematical model describing this battle is given by the coupled differential equations

$$\frac{dR}{dt} = -c_1RB,\ \frac{dB}{dt} = -c_2RB,$$ where R: Red Army, B: Blue Army where c_1 and c_2 are positive constants.

(i) Use the chain rule to find a relationship between R and B, given the initial numbers of soldiers for the two armies are r_0 and b_0, respectively.

(ii) Draw a sketch of typical phase-plane trajectories.

(c) Suppose that soldiers from the red army are visible to the blue army, but soldiers from the blue army are hidden. Thus, the red army is using random firing while the blue army uses aimed firing

(i) Write down appropriate word equations describing the rate of change of the number of soldiers in each of the armies.

(ii) By making appropriate assumptions, obtain two coupled differential equations describing this system.

(iii) Extend the model to include reinforcements if both of the armies receive reinforcements at a constant rate.

(d) The pair of differential equations

$$\frac{dP}{dt} = rP - \gamma PT, \qquad \frac{dT}{dt} = qP,$$

where r, γ and q are positive constants, is a model for a population of microorganisms P, which produces toxins T which kill the microorganisms.

(i) Given that initially there are no toxins and p_0 microorganisms, obtain an expression relating the population density and the amount of toxins.

(ii) Give a sketch of a typical phase-plane trajectory, indicating the direction of movement along the trajectories.

(iii) Using this model, describe what happens to the microorganisms over time.

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1174 B

Unique Paper Code : 32355202

Name of the Paper : GE-2 Linear Algebra

Name of the Course : Generic Elective

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Do any **two** parts from each question.

1. (a) If x and y are vectors in $\mathbb{R}^n$, then prove that $|x \cdot y| \le \|x\| \, \|y\|$.

   Also, verify it for the vectors x = [1, -2, 0, 2, 3], y = [0, -3, 2, -1, -1]. (6)

   (b) Let x and y be non-zero vectors in $\mathbb{R}^n$, then prove that $x \cdot y = \|x\| \, \|y\|$ if and only if y is a positive scalar multiple of x. (6)

P.T.O.

(c) (i) For the vectors a = [6, -5, -2] and b = [-4, -3, 2], find $proj_a b$ and verify that b − $proj_a b$ is orthogonal to a. (3)

(ii) Prove that if (x + y) · (x − y) = 0, then ||x|| = ||y||, where x and y are vectors in $\mathbb{R}^n$. (3)

(d) Use the Gauss-Jordan Method to solve the following system of linear equations

$$x + 2y + z = 8$$
$$2x + 3y + 2z = 14$$
$$3x + 2y + 2z = 13.$$ (6)

2. (a) Determine whether the vector [7,1,18] is in the row space of the matrix

$$A = \begin{bmatrix} 3 & 6 & 2 \\ 2 & 10 & -4 \\ 2 & -1 & 4 \end{bmatrix}$$ (6.5)

(b) Find the characteristic polynomial and eigen value of the matrix

$$A = \begin{bmatrix} 4 & 0 & -2 \\ 6 & 2 & -6 \\ 4 & 0 & -2 \end{bmatrix}$$

Is A diagonalizable? Justify. (6

(c) (i) Show that the set of vectors of the form $[2a - 3b, a - 5c, a, 4c - b, c]$ in $\mathbb{R}^5$ forms a subspace of $\mathbb{R}^5$ under the usual operations. (3)

(ii) Find the eigenspace E_λ corresponding to the eigen value $\lambda = 3$ for the matrix

$$A = \begin{bmatrix} 3 & -1 \\ 0 & 4 \end{bmatrix}$$ (3.5)

(d) Let V be the set $\mathbb{R}^2$ with the operations addition and scalar multiplication for x, y, w, z and a in $\mathbb{R}$ defined by :

$[x, y] \oplus [w, z] = [x + w + 1, y + z - 2]$ and

$a \odot [x, y] = [ax + a - 1, ay - 2a + 2]$.

Prove that V is a vector space over $\mathbb{R}$. Find the zero vector in V and the additive inverse of each vector in V. (6.5)

3. (a) Using rank criterion, check whether the following system is consistent or not?

$$\begin{bmatrix} 1 & -2 & -3 & 4 \\ 4 & -1 & -5 & 6 \\ 2 & 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

If consistent, solve the system. (6)

(b) Use Simplified Span Method to find a simplified general form for all the vectors in span(S), where S = {[1, −1, 1], [2, −3, 3], [0, 1, −1]} is a subset of $\mathbb{R}^3$. (6)

(c) Let B = {[1, −2, 1], [5, −3, 0]} and

S = {[1, −2, 1], [3, 1, −2], [5, −3, 0], [5, 4, −5], [0, 0, 0]} be subsets of $\mathbb{R}^3$.

(i) Show that B is a maximal independent subset of S.

(ii) Calculate dim(span(S)).

(iii) Does span(S) = $\mathbb{R}^3$? Justify. (6)

(d) Use the Independence Test Method to show that the subset

S = $\{x^2 + x + 1,\ x^2 - 1,\ x^2 + 1\}$ of P_2 is linearly independent. (6)

4. (a) (i) Let f: $\mathbb{R}^2 \to \mathbb{R}^2$ be given by

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + ky \\ y \end{bmatrix}$$

Show that f is a linear operator.

(ii) Let V be a vector space, and let x ≠ 0 be a fixed vector in V. Prove that the translation function f: V → V given by f(v) = v + x is not linear. (6.5)

(b) Let S = {[1,2], [0,1]} and T = {[1,1], [2,3]} be bases for $\mathbb{R}^2$. Let v = [1,5].

(i) Find the coordinate vector of v with respect to the basis T.

(ii) What is the transition matrix $P_{S \leftarrow T}$ from the basis T to the basis S.

(iii) Find the coordinate vectors of v with respect to S using $P_{S \leftarrow T}$. (6.5)

(c) Suppose L: $\mathbb{R}^3 \to \mathbb{R}^3$ is a linear operator and L([1,0,0]) = [−3,2,4], L([0,1,0]) = [5,−1,3] and L([0,0,1]) = [−4,0,−2]. Find L([6,2,−7]). Find L([x, y, z]), for any [x, y, z] ∈ $\mathbb{R}^3$. (6.5)

(d) Let L: $\mathbb{R}^3 \to \mathbb{R}^2$ defined by

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -2 & 0 & 3 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

be a linear transformation and B = {[1, −3,2], [−4,13,−3], [2,−3,20]} and C = {[−2,−1], [5,3]} be bases for $\mathbb{R}^3$ and $\mathbb{R}^2$, respectively. Find the matrix A_{BC} of L w.r.t. B and C. (6.5)

5. (a) Consider the linear transformation L: $\mathbb{R}^3 \to \mathbb{R}^2$ given by

L([x, y, z]) = [x + y, y + z]. Show that L is onto but not one to one. (6)

(b) Find the minimum distance from the point P(1,4,−2) to the subspace W = span {[x,y,z]: −2x + 5y − z = 0} of $\mathbb{R}^3$. (6)

(c) Find a least squares solution for the linear system AX = B, where

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \\ 4 & 3 \end{bmatrix}, \ B = \begin{bmatrix} 12 \\ 15 \\ 14 \end{bmatrix} \quad (6)$$

(d) Consider a polygon associated with 2 × 5 matrix

$$\begin{bmatrix} 8 & 8 & 6 & 8 & 10 & 10 \\ 6 & 8 & 10 & 12 & 10 & 6 \end{bmatrix}.$$ Use ordinary coordinates in $\mathbb{R}^2$ to find the new vertices after performing each indicated operation :

(i) translation along the vector [12,6].

(ii) rotation about the origin through θ = 90°.

(iii) reflection about the line y = −3x.

(iv) scaling about the origin with scale factors of 1/2 in the x-direction and 4 in the y-direction. (6)

6. (a) Let L : $\mathbb{R}^3 \to \mathbb{R}^3$ be the Linear Operator given by

$$L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} -4 & 2 & 6 \\ 1 & 1 & -3 \\ 2 & 0 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Find the basis for Ker(L) and basis for Range(L). Verify Dimension Theorem. (6.5)

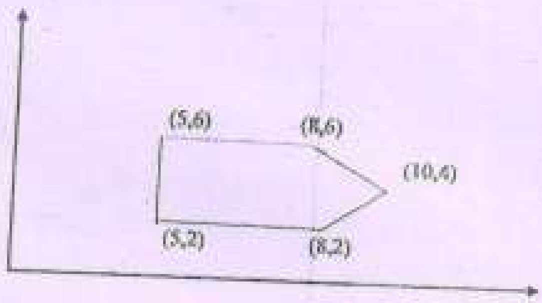
(b) For the subspace W = {[x, y, z]: −x + 4y − 2z = 0} of $\mathbb{R}^3$, find $W^\perp$, the orthogonal complement of W. Verify $\dim(W) + \dim(W^\perp) = \dim(\mathbb{R}^3)$. (6.5)

(c) Verify that the given ordered basis B is orthonormal. Hence, for the given v, find $[v]_B$

where $v = [-2,3]$, $B = \left\{ \left[\frac{-\sqrt{3}}{2}, \frac{1}{2} \right] \left[\frac{1}{2}, \frac{\sqrt{3}}{2} \right] \right\}$.

(6.5)

(d) For the given graphic, use homogeneous coordinates to find the new vertices after performing a scaling about (2,2) with scale factor of 2 in x-direction and 3 in y-direction. Sketch the final figure resulting from the movement.



(6.5)

May 2022 (7)

[This question paper contains 6 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 738 B

Unique Paper Code : 32351201

Name of the Paper : BMATH203 – Real Analysis

Name of the Course : B.Sc. (H) Mathematics

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All Questions are Compulsory.
3. Attempt any two parts from each question.
4. All Questions are of equal marks.

1. (a) State the completeness property of $\mathbb{R}$, hence show that every non-empty set of real numbers which is bounded below, has an infimum in $\mathbb{R}$.

P.T.O.

(b) Show that if A and B are bounded subsets of $\mathbb{R}$, then $A \cup B$ is a bounded set and $\sup (A \cup B) = \max \{\sup A, \sup B\}$.

(c) State and prove nested interval property.

(d) Define an open set and closed set in $\mathbb{R}$.

Show that if $a, b \in \mathbb{R}$, then the open interval (a, b) is an open set.

Is a closed interval a closed set ?

2. (a) Let S be a bounded set in $\mathbb{R}$ and let S_0 be a non-empty subset of S. Show that

$$\inf S \leq \inf S_0 \leq \sup S_0 \leq \sup S$$

(b) State Archimedean property. Hence, prove that if

$S = \left\{\frac{1}{n}, n \in N\right\}$ then $\inf S = 0$.

(c) If $S \subseteq \mathbb{R}$ is non empty. Show that S is bounded if and only if there exists a Closed bounded interval I such that $S \subseteq I$.

(d) If $x, y, z \in \mathbb{R}$ and $x \leq z$. Show that $x \leq y \leq z$ if and only if $|x - y| + |y - z| = |x - z|$. Interpret this geometrically.

3. (a) Prove that a convergent sequence of real numbers is bounded.

Is the converse true? Justify.

(b) Let (x_n) be a sequence of positive real numbers

such that $\lim\limits_{n\to\infty}\left(\frac{x_{n+1}}{x_n}\right) = L$ exists. If $L < 1$, then

(x_n) converges and $\lim\limits_{n\to\infty}(x_n) = 0$.

(c) Prove that if $C > 0$, then $\lim\limits_{n\to\infty}\left(C^{1/n}\right) = 1$.

(d) Let $x_1 > 1$ and $x_{n+1} = 2 - \frac{1}{x_n}$ for $n \in N$. Show

that (x_n) is bounded and monotone. Also find the limit.

4. (a) Let $X = (x_n)$ and $Y = (y_n)$ be sequences of real numbers that converge to x and y respectively. Then the product sequence X.Y converges to x.y.

(b) Let $X = (x_n)$ be a bounded sequence of real numbers and let $x \in R$ have the property that every convergent subsequence of X converges to x. Then the sequence X is convergent to x.

(c) Discuss the convergence of the sequence (x_n),

where $x_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots\cdots + \frac{1}{2n}$ for $n \in N$.

(d) Use the definition of the limit of the sequence to find the following limits

(i) $\lim_{n\to\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$

(ii) $\lim_{n\to\infty} \left(\frac{3n+1}{2n+5}\right)$

5. (a) Prove that a necessary condition for the convergence of an infinite series Σa_n is $\lim_{n\to\infty} a_n = 0$. Is the condition sufficient? Justify with the help of an example.

(b) Prove that the geometric series $1 + r + r^2 + \cdots$ converges for $0 \leq r \leq 1$ and diverges for $r \geq 1$.

(c) Test for convergence, the following series :

(i) $\frac{1}{5} + \frac{2!}{5^2} + \frac{3!}{5^3} + \cdots$

(ii) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n-1}}{n}$

(d) Prove that the series $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$ converges if and only if $-1 \leq x \leq 1$.

6. (a) State and prove Cauchy's n^{th} root test for positive term series.

(b) Prove that the series $1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} \cdots$ converges for $p > 1$ and diverges for $p \leq 1$.

(c) Test for convergence, the following series :

(i) $\sum_{n=1}^{\infty} \left[\sqrt[3]{n^3+1} - n\right]$

(ii) $\sum_{n=1}^{\infty} 2^{-n-(-1)^n}$

(d) Prove that every absolutely convergent series is convergent. Show that the series $\sum(-1)^n \frac{n+2}{2^n+5} x^n$ converges for all the real values of x.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1075 **A**

Unique Paper Code : 32353401

Name of the Paper : SEC-2 Computer Algebra Systems and Related Softwares

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : IV

Duration : 2 Hours Maximum Marks : 38

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. There are **six** questions in all.
3. **Q1** and **Q4** are compulsory. Attempt any **two** questions from the rest.

UNIT – 1 (CAS)

Note : The answers should be written in only **one** of the CAS:Mathematica/MATLABMaxima/Maple or any other.

P.T.O.

1. Fill in the blanks : **(Any ten)** (10)

(a) Numerical value of $\frac{5}{7}$ up to 10 places are given by command ______

(b) Command for Square of a previous output is ______

(c) Command for 100! is ______

(d) The output for ArcTan[$\frac{1}{4}$] is ______

(e) The command for 7 mod 3 is ______

(f) The command for numeric value of $\sqrt{3}\sqrt{5}$ is ______

(g) Command for numerical approximation to 13^{20} with 15 significant digit is ______

(h) Command for the Table of the squares of the first five positive whole numbers is ______

(i) ______ is used to plot an implicitly defined function.

(j) The command ______ will produce a formatted rectangular array with brackets on the sides.

(k) The symbol ______ will simply multiply corresponding entries in the two matrices.

(l) The command for natural logarithm is ______

2. Answer any **two** parts from the following : (4.5×2=9)

(a) Define the function f(x) = cos 3x +sin 3x. Find its derivative and integral between the limits [0,π/3] and write the commands for the same.

(b) Write command for sketching the curve :

x = 1 + Sin(t)

y = 2Cos(2t), {t, 0, 2π}

(c) Let $A=\begin{bmatrix}2 & 4 & 5\\ 3 & 1 & 8\\ 7 & 3 & 2\end{bmatrix}$ $B=\begin{bmatrix}7 & 5 & 1\\ 1 & 4 & 2\\ 3 & 1 & 2\end{bmatrix}$

Write command for generating

(i) Matrix (A + B).

(ii) Matrix Multiplication of A and B.

(iii) Pointwise Multiplication of A and B.

(d) Write command for generating graph of the surface :

$z=e^{-\left(\frac{x^2}{2}+\frac{y^2}{2}\right)}$ for $-5 \le x,\ y \le 5$.

3. Answer any **two** parts from the following : (4.5×2=9)

(a) Write a command to find the adjoint of matrix

$$A=\begin{pmatrix}1 & 1 & 0\\ 0 & 1 & 1\\ 1 & 2 & 3\end{pmatrix}$$

using determinant and inverse of A and check your answer by finding co-factor matrix of A.

(b) Write output of the following command

s = SparseArray[Table[{i+1,2^i} → i^2, {i,3}]]

and also give the command which describes positions of non zero elements in s.

(c) Let S = {v1, v2, v3} where v1 = {1, 2, 3}, v2 = {1, -1, 1}, v3 = {4, 5, 9}

Write commands to

(i) Find nullity of the matrix whose columns are given by vectors in S.

(ii) Find whether the vector b = {-1, 2, 5} lies in the span of S.

(d) For the matrix

$$\begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 2\\ 2 & 3 & 1\end{pmatrix}$$

Write commands

(i) To find eigenvalues and eigenvectors.

(ii) To diagonalize the matrix.

UNIT – 2 (R Programming)

4. State whether the following statements are True or False : **(Any ten)** (10)

(a) *seq (2,10,1)* command is used to form a vector 2,3,4,5,6,7,8,9,10.

(b) The command *length(frame)* gives the number of items in the data frame.

(c) The apply() command enables you to apply a function to columns only of a matrix or data frame.

(d) The command hist() is used for recalling all previous commands.

(e) The data must be all numbers or all characters to form a matrix.

(f) The NA is a special R object and always used as a character.

(g) The quantile () command is to produce 25%, 50%,75%, 100%.

(h) To access the elements of data objects ,you can use $.

(i) The data frame can not handle mixed data.

(j) The length of the following vector is 7:

week={Sun,Mon,Tue,Wed,Thu,Fri,Sat,NA}

(k) To combine data samples, you can use cbind() command.

(l) To examine the mean of the third row of a matrix named birds, you can use the command means(birds,[3,]).

5. Answer any **two** parts from the following : (4.5×2=9)

(a) (i) Consider the data 1=5,8,3,1,9,2,4,4,7,3. Write a command to remove the values 1,9,2 from the data1

(ii) For the samples:

sample1: 5, 6, 9, 12, 8
sample2: 7, 9, 13, 10

Write the command to make a data frame.

(b) (i) Write a command to create a pie chart with labels for the following datas :

data1: 3 5 7 5 3 2 6 8 5 6 9 8
data2: "Jan" "Feb" "Mar" "Apr" "May" "Jun" "Jul" "Aug" "Sep" "Oct" "Nov" "Dec".

(ii) Find the minimum value of data1.

(c) (i) Using scan command enter the following data: vegetables={carrot, onion, peas, brinjal}

(ii) Put the items in alphabetical order using a command.

(d) (i) Give a command to read a file of data from a disk.

(ii) Write any command that produces multiple values as a result of the data.

6. Answer any **two** parts from the following : (4.5×2=9)

(a) Write commands to evaluate

(i) product of square roots of 30 and 50.

(ii) sum of 3^{rd} and 5^{th} power of pi.

(iii) sum of squares of positive divisors of 3.

(b) Write commands to

(i) read character data from the file 'names.csv'.

(ii) enter the names of months of the year having 31 days.

(iii) make a new vector from the vectors obtained in part (i) and (ii).

(c) Write commands to

(i) get the list of objects that end with 'e'.

(ii) to remove the list of all objects having letter 'b' in their name.

(iii) to get the list of all objects starting with either 'a' or 'e'

(d) Write commands to

(i) convert a vector v containing names of days of the week into numeric vector w.

(ii) get the structure of v and w.

(iii) get the structure of all available objects with 'data' in their name.

[This question paper contains 6 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : **1943** **A**

Unique Paper Code : 32355402

Name of the Paper : GE-4 Numerical Methods

Name of the Course : **Generic Elective CBCS (LOCF)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory and carry equal marks.

1. (a) Find the interval in which the smallest positive root of the equation $x^3 - x - 4 = 0$ lies. Perform three iterations of the bisection method to determine the root of this equation.

P.T.O.

(b) Perform three iterations of the secant method to find a root of the equation $xe^x = \cos x$ by taking $p_0 = 0$, $p_1 = 1$.

(c) Perform four iterations of the Newton-Raphson's method to obtain the approximate value of $(17)^{\frac{1}{3}}$ starting with the initial approximation $x_0 = 2$.

(d) Define Floating-point representation, Truncation error, and Global error with examples.

2. (a) Define the order of convergence of an iterative method. Determine the order of convergence of the Regula-Falsi method.

(b) Perform three iterations of the bisection method to find the smallest positive root of the equation $x^3 - 4x - 9 = 0$.

(c) Perform three iterations of the method of false position, to find the fourth root of 32.

(d) Perform three iterations of the Newton Raphson method to find the root of $f(x) = x \sin x + \cos x = 0$, assuming that the root is near $x = \pi$.

(a) Generate the forward difference table for the data

x	0	0.2	0.4	0.6	0.8
f(x)	0.12	0.46	0.74	0.9	1.2

Hence interpolate the values of $f(0.1)$ by using Gregory Newton forward differences Interpolation formulae.

(b) Define the average difference operator and the central difference operator. Prove that $\mu = \left(1 + \frac{\delta^2}{4}\right)^{\frac{1}{2}}$.

(c) Given the following system of equations

$$x_1 + x_2 + x_3 = 1$$
$$4x_1 + 3x_2 - x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 4$$

Perform three iterations of the Gauss-Jacobi method starting with $X^{(0)} = (1,1,1)$.

(d) Find the unique polynomial of degree 3 or less such that $f(0) = -1$, $f(1) = 0$, $f(2) = 15$, $f(3) = 80$ using the Newton interpolation. Interpolate at $x = 1.5$.

(a) Perform three iterations of the Gauss-Seidel method starting with $X^{(0)} = (1,1,0)$ to solve the following system of equations:

$$10x_1 + x_2 + 4x_3 = 31$$
$$x_1 + 10x_2 - 5x_3 = -23$$
$$3x_1 - 2x_2 + 10x_3 = 38.$$

(b) Solve the following system using the Gauss-Jordan method.

$$x + y + z = 7$$
$$x + 2y + 3z = 16$$
$$x + 3y + 4z = 22.$$

(c) Calculate the second order divided difference of $\frac{1}{x^2}$ based on the points x_0, x_1, x_2.

(d) Determine the Lagrange form of the interpolating polynomials for the following data set

x	-1	0	1	2
y=f(x)	5	1	1	11

Hence estimate the value of f(1.5).

5. (a) For the following data, find f'(2.3) and f''(2.3) b using Central difference formulas

$$f'(x_i) = \frac{f(x_i + h) - f(x_i - h)}{2h}$$

$$f''(x_i) = \frac{f(x_i + h) - 2f(x_i) + f(x_i - h)}{h^2}$$

x	2	2.3	2.6
y = f(x)	0.6932	0.7885	0.9556

(b) The following table of values is given :

x	-1	1	2	3	4	5	7
y=f(x)	1	1	16	81	256	625	2401

Find f'(3) by Richardson Extrapolation with h = 4, h = 2 and h = 1 using the following approximate formula

$$f'(x_i) = \frac{f(x_i + h) - f(x_i - h)}{2h}$$

(c) Find an approximate value of the integral

$$I = \int_0^1 \frac{1}{1 + x}\, dx$$

Using: (i) Trapezoidal Rule, (ii) Composite Trapezoidal rule for n = 4.

Also calculate the error in each case.

P.T.O.

(d) Find an approximate value of the integral

$$I = \int_1^2 \frac{1}{1+x^2}\,dx$$

Using (i) Simpson's rule, (ii) Composite Simpson's rule for n=6.

Also calculate the error in each case.

6. (a) Approximate the value of $\pi/4$ by using Simpson's $1/3^{rd}$ rule.

(b) Apply Euler's method to approximate the solution of the Initial value Problem (IVP)

$$\frac{dy}{dx} = 2 + \frac{3y}{x},\ 1 \le x \le 6,\ y(1) = 1, \text{ using 5 steps.}$$

(c) Apply Modified Euler's method to approximate the solution of the IVP and calculate y(0.3) by using h=0.1

$$\frac{dy}{dx} = 1 + xy,\ 0 \le x \le 1,\ y(0) = 2.$$

(d) Use the Midpoint method to approximate the solution of

$$\frac{dy}{dx} = 2x + 3y,\quad y(0) = 0$$

with h=0.1. Determine y(0.2) and y(0.3).

(200

(54)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper :	**1150**	**A**
Unique Paper Code	: 32351401_LOCF	
Name of the Paper	: BMATH-408 Partial Differential Equation	
Name of the Course	: **CBCS B.Sc. (H) Mathematics**	
Semester	: IV	

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **four** questions.
3. **All** questions carry equal marks.

SECTION – I

Attempt any **two** parts out of the following.

Marks of each part are indicated.

1. (a) Define the following with one example each : (6)

P.T.O.

(i) Quasi-linear first order partial differential equation (PDE).

(ii) Semi-linear first order PDE.

(iii) Linear first order PDE.

State whether the following first order PDE is quasi-linear, semi-linear, linear or non-linear :

$$\left(xy^2\right)u_x - \left(yx^2\right)u_y = u^2(x^2 - y^2)$$

Justify.

(b) Solve the Cauchy problem (6)

$$uu_x + u_y = 1$$

such that $u(s,0) = 0, x(s,0) = 2s^2,$

$$y(s,0) = 2s, s > 0.$$

(c) Obtain the solution of the pde (6)

$$x\left(y^2 + u\right)u_x - y\left(x^2 + u\right)u_y = (x^2 - y^2)u,$$

with the data u(x,y) = 1 on x + y = 0.

(d) Apply $\sqrt{u} = v$ and $v(x, y) = f(x) + g(y)$ to solve

$$x^4 u_x^2 + y^2 u_y^2 = 4u.$$ (6)

2. Attempt any **two** parts out of the following :

(a) Apply the method of separation of variables $u(x,y) = f(x)g(y)$ to solve

$$y^2 u_x^2 + x^2 u_y^2 = (xyu)^2$$

such that $u(x,0) = 3e^{\frac{x^2}{4}}$. (6.5)

(b) Find the solution of the equation (6.5)

$$yu_x - 2xyu_y = 2xu$$

with the condition $u(0,y) = y^3$.

(c) Reduce into canonical form and solve for the general solution (6.5)

$$u_x - yu_y - u = 1.$$

(d) Derive the one-dimensional heat equation :

$$u_t = \kappa u_{xx},$$

where κ is a constant. (6.5)

P.T.O.

SECTION - II

3. Attempt any **two** parts out of the following :

(a) Find the characteristics and reduce the equation

$u_{xx} - (\sec h^4 x) u_{yy} = 0$ into canonical form. (6)

(b) Find the characteristics and reduce the equation

$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} + xy u_x + y^2 u_y = 0$

into canonical form. (6)

(c) Transform the equation $u_{xx} - u_{yy} + 3u_x - 2u_y + u = 0$ to the form $v_{\xi\eta} = cv$, c=constant, by introducing the new variable $v = ue^{-(a\xi + b\eta)}$, where a and b are undetermined coefficients. (6)

(d) Use the polar co-ordinates r and θ (x=r cosθ, y=r sinθ) to transform the Laplace equation $u_{xx} + u_{yy} = 0$ into polar form. (6)

4. Attempt any **two** parts out of the following :

(a) Find the D'Alembert solution of the Cauchy problem for one dimensional wave equation given by

$$u_{tt} - c^2 u_{xx} = 0, x \in R, t > 0$$
$$u(x,0) = f(x), x \in R,$$
$$u_t(x,0) = g(x), x \in R.$$

(6.5)

(b) Solve (6.5)

$$y^3 u_{xx} - y u_{yy} + u_y = 0,$$
$$u(x,y) = f(x) \text{ on } x + \frac{y^2}{2} = 4 \text{ for } 2 \le x \le 4,$$
$$u(x,y) = g(x) \text{ on } x - \frac{y^2}{2} = 0 \text{ for } 0 \le x \le 2,$$
$$\text{with } f(2) = g(2).$$

(c) Determine the solution of initial boundary value problem

$$u_{tt} = 16 u_{xx}, \quad 0 < x < \infty, t > 0$$
$$u(x,0) = \sin x, \quad 0 \le x < \infty,$$
$$u_t(x,0) = x^2, \quad 0 \le x < \infty,$$
$$u(0,t) = 0, \quad t \ge 0.$$

(6.5)

(d) Determine the solution of initial boundary value problem (6.5)

$$u_{tt} = 9u_{xx}, \quad 0 < x, \infty, t > 0,$$
$$u(x,0) = 0, \quad 0 \le x < \infty,$$
$$u_t(x,0) = x^3, \quad 0 \le x < \infty$$
$$u_x(0,t) = 0, \quad t \ge 0.$$

SECTION - III

5. Attempt any **two** parts out of the following :

(a) Determine the solution of the initial boundary-value problem by method of separation of variables

$$u_{tt} = c^2 u_{xx}, \quad 0 < x < l, \; t > 0$$
$$u(x,0) = \begin{cases} h\,x/a, & 0 \le x \le a \\ h\,(l-x)/(l-a), & a \le x \le l \end{cases},$$
$$u_t(x,0) = 0, \qquad 0 \le x \le l,$$
$$u(0,t) = 0 = u(l,t) = 0 \qquad t \ge 0$$

(6.5)

(b) Obtain the solution of IBVP (6.5)

$$u_t = u_{xx}, \quad 0<x<2, \; t>0,$$
$$u(x,0)=x, \; 0 \le x \le 2,$$
$$u(0,t)=0, \quad u_x(2,t)=1, \qquad t \ge 0,$$

(c) Determine the solution of the initial-value problem (6.5)

$$u_{tt} = c^2 u_{xx} + x^2,$$
$$u(x,0) = x, \qquad 0 \le x \le 1,$$
$$u_t(x,0) = 0, \qquad 0 \le x \le 1,$$
$$u(0,t) = 0, u(1,t) = 0, \qquad t > 0.$$

(d) Determine the solution of the initial-value problem (6.5)

$$u_t = k u_{xx}, \qquad 0 < x < 1, t > 0,$$
$$u(x,0) = x(1-x), \qquad 0 \le x \le 1$$
$$u(0,t) = t, \quad u(1,t) = \sin t, \quad t > 0.$$

6. Attempt any **two** parts out of the following :

(a) Determine the solution of the initial boundary-value problem by method of separation of variables (6)

$$u_{tt} = c^2 u_{xx}, 0 < x < a, \; t > 0$$
$$u(x,0) = 0, \; 0 \le x \le a,$$
$$u_t(x,0) = \begin{cases} \frac{v_0}{a} x, & 0 \le x \le a \\ v_0\,(l-x)/(l-a), & a \le x \le l \end{cases}$$
$$u(0,t) = 0 = u(a,t) = 0, \qquad t \ge 0.$$

(b) Find the temperature distribution in a rod of length l. The faces are insulated, and the initial temperature distribution is given by x (l - x). (6)

(c) Establish the validity of the formal solution of the initial boundary – value problem (6)

$$u_t = ku_{xx}, \quad 0 \le x \le l, \; t > 0,$$
$$u(x,0) = f(x), \qquad 0 \le x \le l,$$
$$u(0,t) = 0, \qquad t > 0,$$
$$u_x(1,t) = 0, \qquad t > 0.$$

(d) Prove the uniqueness of the solution of the problem : (6)

$$u_{tt} = c^2 u_{xx}, \quad 0 < x < l, \; t > 0,$$
$$u(x,0) = f(x), \; 0 \le x \le l,$$
$$u_t(x,0) = g(x), 0 \le x \le l,$$
$$u(0,t) = u(0,t) = 0, \; t > 0.$$

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1505 **A**

Unique Paper Code : 42354401

Name of the Paper : Real Analysis

Name of the Course : **B.Sc. Mathematical Sciences/ B.Sc. (Prog.)**

Semester : IV

Duration : 3.5 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt any **two** parts from each question.

1. (a) Show that a countable union of countable sets is countable. Deduce that the set $N \times N$ is countable.

   (b) State and prove the Archimedean property of real numbers.

   (c) Let S be a non-empty bounded set in R.
   Show that

$$Sup(a\,S) = a\,SupS \;\; if\; a > 0$$
$$\& \;\; Inf(a\,S) = a\,Sup\,S \;\; if\; a < 0$$

P.T.O.

(d) Find all $x \in R$ that satisfy the following inequalities :

(i) $|x-1| > |x+1|$

(ii) $|x| + |x+1| < 2$

(6, 6)

2. (a) Let A and B be two non-empty subsets of R that satisfy the property:
$a \leq b \; \forall \, a \in A \,\&\, \forall \, b \in B$
show that $Sup\, A \leq Inf\, B$.

(b)Use the definition of the limit of a sequence to establish the following limits

(i) $Lt\left(\frac{2n+1}{2n+5}\right) = \frac{2}{2}$

(ii) $Lt\left(\sqrt{n+1} - \sqrt{n}\right) = 0$

(c) If $Lt\,(x_n) = x > 0$, show that $\exists$ a natural no. k : if $n \geq k$, then
$\frac{1}{2}x < x_n < 2x$

(d) Let $< x_n >$ be a sequence of positive real numbers such that
$Lt\left(x_n^{1/n}\right) = l < 1$, show that $Lt\,(x_n) = 0$.

(6, 6)

3. (a) State and prove Monotone Convergence theorem.

(b) Establish the convergence or the divergence of the sequence $< x_n >$, where
$x_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}, n \in N$.

(c) State Cauchy Convergence Criteria for sequences.
Show that the sequence $< x_n >$, where
$x_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{n}$ is a divergent sequence.

(d) State and prove the necessary condition for the convergence of an infinite series.

Is the condition sufficient ? Justify the answer.

(6.5, 6.5)

4. (a) Test for the convergence and absolute convergence of the series
$1 - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{7}} + \cdots$

(b) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is a divergent series when $p < 1$.

(c) Test for the convergence of the following series :

(i) $\sum_{n=1}^{\infty} \left(\sqrt{n^3+1} - n^{3/2} \right)$

(ii) $\sum_{n=1}^{\infty} \frac{1}{n^n}$.

(d) Test the series
$1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \cdots$ for convergence, for all positive values of x .

(6, 6)

5. (a) Prove that $\lim_{n \to \infty} \frac{nx}{1+n^2x^2} = 0 \; \forall \, x \in R$.
Also prove that $< \frac{nx}{1+n^2x^2} >$ is not uniformly convergent on $[0, \infty[$, but is uniformly convergent on $[\, a, \infty [\, , a > 0$.

(b) Discuss the convergence and uniform convergence of the series $\sum f_n(x)$ where $f_n(x)$ is given by $(x^2 + n^2)^{-1}$.

(c) Determine the radius of convergence and exact interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (x-1)^n$.

(d) State and prove Weierstrass M – test for uniform convergence of a series of functions $\sum f_n(x)$, also test the uniform convergence of $\sum \frac{\sin(x^2+n^2x)}{n(n+1)} \; \forall \, x \in R$.

(6.5, 6.5)

6. (a) Define exponential function in terms of power series. Prove that

(i) $E'(x) = E(x) \, \forall \, x \in R$

(ii) $E(x+y) = E(x)\,E(y) \; \forall \, x, y \in R$.

(iii) $E(r) = e^r \; \forall \, r \in Q$

where E denotes exponential function.

(b) Let f be a bounded real function defined on $[a, b]$. Let P be any partition of $[a, b]$. Define the upper and lower sums of f over P and show that
$m(b-a) \leq L(P, f) \leq U(P, f) \leq M(b-a)$

(c) Show that the function $f(x) = x^2 + x$ is integrable on $[0, 1]$ and find $\int_0^1 f(x)$, using Sequential Criteria .

(d) Let f be a bounded real function defined on $[a, b]$. Let P be any partition of $[a, b]$. Define the upper and lower sums of f over P and show that

$$m(b-a) \leq L(P, f) \leq U(P, f) \leq M(b-a)$$

where m and M are bounds of f over $[a, b]$. (6.5, 6.5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1377 **A**

Unique Paper Code : 32351402

Name of the Paper : BMATH-409: Riemann Integration and Series of Functions

Name of the Course : **B.Sc. (H) Mathematics**

Semester : IV

Duration : 3 hours + 30 minutes Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory.

1. (a) Let f be a bounded function on [a, b]. Define integrability of f on [a, b] in the sense of Riemann. (6)

P.T.O.

(b) Prove that every continuous function on [a, b] is integrable. Discuss about the integrability of discontinuous functions. (6)

(c) Let $f(x) = \sin\frac{1}{x}$ for $x \neq 0$ and $f(0) = 0$. Show that f is integrable on [-1, 1], Show that $\left|\int_{-1}^{1} f(t)\,dt\right| \leq 2$. (6)

(d) Let f(x) = x for rational x; and f(x) = 0 for irrational x. Calculate the upper and lower Darboux integrals of f on the interval [0, b]. Is f integrable on [0, b]? (6)

2. (a) State Fundamental Theorem of Calculus II. Use it to calculate

$$\lim_{x\to 0}\frac{1}{x}\int_0^x e^{t^2}\,dt.$$ (6.5)

(b) Let f be defined as (6.5)

$$f(t)=\begin{cases}0, & t<0\\ t, & 0\leq t\leq 1\\ 4, & t>1\end{cases}$$

(i) Determine the function $F(x)=\int_0^x f(t)dt$.

(ii) Sketch F. Where is F continuous?

(iii) Where is F differentiable? Calculate F' at points of differentiability.

(c) State and prove Intermediate value theorem for Integral Calculus. Give an example to show that condition of continuity of the function cannot be relaxed. (6.5)

(d) Let f be a continuous function on $\mathbb{R}$. Define

$$G(x)=\int_0^{\sin x} f(t)dt \text{ for } x \in \mathbb{R}.$$

Show that G is differentiable on $\mathbb{R}$ and compute G'. (6.5)

3. (a) Let $\beta(p, q)$ (where $p, q > 0$) denotes the beta function, show that

$$\beta(p,q)=\int_{0^+}^{\infty}\frac{u^{p-1}}{(1+u)^{p+q}}du=\int_{0^+}^{1}\frac{v^{p-1}+v^{q-1}}{(1+v)^{p+q}}dv. \quad (6)$$

(b) Determine the convergence and divergence of the following improper integrals

(i) $\int_0^1 \frac{dx}{x(\ln x)^2}$

(ii) $\int_{-1}^{\infty} \frac{dx}{x^2+4x+6}$ (6)

(c) Define Improper Integral of type II.

Show that the improper integral $\int_1^{\infty} \frac{dx}{x^p}$ converges iff $p > 1$. (6)

(d) Show that the improper integral $\int_{\pi}^{\infty} \frac{\text{Sin } x}{x} dx$ is convergent but doesn't converge absolutely. (6)

4. (a) Let $< f_n >$ be a sequence of integrable functions on [a,b] and suppose that $< f_n >$ converges uniformly on [a, b] to f. Show that is f is integrable. (6)

(b) Define

(i) pointwise convergence of sequence of functions

(ii) uniform convergence of a sequence of functions

(iii) If $A \subseteq \mathbb{R}$ and $\emptyset: A \to \mathbb{R}$ then define uniform norm of $\emptyset$ on A. (6)

(c) (i) Discuss the pointwise and uniform convergence of $f_n(x) = \frac{x}{n}$ for $x \in \mathbb{R}$, $n \in N$.

(ii) Show that the sequence $< f_n >$ where $f_n(x) = \frac{n}{x+n}$, $x \geq 0$ is uniformly convergent in any finite interval. (6)

(d) (i) Show that the sequence $< f_n >$ where $f_n(x) = \frac{\sin nx}{\sqrt{n}}$ uniformly convergent on $[0, \pi]$.

(ii) Discuss the pointwise and uniform convergence of the sequence $g_n(x) = x^n$ for $x \in \mathbb{R}$, $n \in N$. (6)

5. (a) (i) State and prove the Cauchy Criteria for uniform convergence of series. (3.5)

(ii) Show that the series $\sum_0^\infty (1-x)x^n$ is not uniformly convergent on [0, 1] (3)

(b) Show that $\sum \frac{(-1)^n}{n^p} \frac{x^{2n}}{(1+x^{2n})}$ converges absolutely and uniformly for all values of x if $p > 1$.

(c) Is the sequence $< f_n >$ where $f_n = \frac{\sin(nx+n)}{n}$, uniformly convergent on $\mathbb{R}$? Justify. (6.5)

(d) If f_n is continuous on $D \subseteq \mathbb{R}$ to $\mathbb{R}$ for each $n \in N$ and $\sum f_n$ converges to f uniformly on D then prove that f is continuous on D. (6.5)

6. (a) Find the radius of convergence and exact interval of convergence of the following power series :

(i) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}(x+1)^n$

(ii) $\sum_{n=0}^{\infty} x^{n!}$ (6.5)

(b) Let $f(x) = \sum_{n=0}^{\infty} a_n x^n$ has radius of convergence $R > 0$. Show that the function f is differentiable on (–R, R) and

$f'(x) = \sum_{n=1}^{\infty} na_n x^{n-1}$ for $|x| < R$. (6.5)

(c) Show that

(i) $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ----$ for $|x| < 1$

(ii) $\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} -----$. (6.5)

(d) Let $f(x)=\sum_{n=0}^{\infty} a_n x^n$ be a power series with radius of convergence $R > 0$. If $0 < R_1 < R$, show that the power series converges uniformly on $[-R_1, R_1]$. Also, show that the sum function f(x) is continuous on the interval $(-R, R)$. (6.5)

[This question paper contains 6 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1395 **A**

Unique Paper Code : 32351403

Name of the Paper : BMATH-410 – Ring Theory and Linear Algebra – I

Name of the Course : **CBCS (LOCF) B.Sc. (H) Mathematics**

Semester : IV

Duration : 3:30 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory.

1. (a) Define zero divisors in a ring. Let R be the set of all real valued functions defined for all real numbers under function addition and multiplication. Determine all zero divisors of R. (6½)

P.T.O.

(b) What is nilpotent element? If a and b are nilpotent elements of a commutative ring, show that a + b is also nilpotent. Give an example to show that this may fail if the ring R is not commutative. (6½)

(c) Let R be a commutative ring with unity. Prove that U(R), the set of all units of R, form a group under multiplication of R. (6½)

(d) Determine all subrings of $\mathbb{Z}$, the set of integers. (6½)

2. (a) Define centre of a ring. Prove that centre of a ring R is a subring of R. (6)

(b) Suppose R is a ring with $a^2 = a$, for all $a \in R$. Show that R is a commutative ring. (6)

(c) Show that any finite field has order p^n, where p is prime. (6)

(d) Let R be a ring with unity 1. Prove that if 1 has infinite order under addition, then CharR = 0, and if 1 has order n under addition, then CharR = n. (6)

3. (a) Let R be a commutative ring with unity and let A be an ideal of R. Then show that R/A is a field if and only if A is maximal ideal. (6½)

(b) Prove that $I = \langle 2 + 2i \rangle$ is not prime ideal of $\mathbb{Z}[i]$. How many elements are in $\frac{\mathbb{Z}[i]}{I}$? What is the characteristic of $\frac{\mathbb{Z}[i]}{I}$? (6½)

(c) In $\mathbb{Z}[x]$, the ring of polynomials with integer coefficients, let $I = \{f(x) \in \mathbb{Z}[x] \mid f(0) = 0\}$. Prove that I is not a maximal ideal. (6½)

(d) Let $\mathbb{R}[x]$ denote the ring of polynomials with real coefficients and let $\langle x^2 + 1 \rangle$ denote the principal ideal generated by $x^2 + 1$. Then show that

$$\frac{\mathbb{R}[x]}{\langle x^2+1 \rangle} = \{g(x) + \langle x^2+1 \rangle \mid g(x) \in \mathbb{R}[x]\} = \{ax + b + \langle x^2+1 \rangle \mid a, b \in \mathbb{R}\}$$

(6½)

4. (a) If R is a ring with unity and the characteristic of R is $n > 0$, then show that R contains a subring isomorphic to $\mathbb{Z}_n$ and if the characteristic of R is 0 then R contains a subring isomorphic to $\mathbb{Z}$. (6)

(b) Determine all ring homomorphism from $\mathbb{Z}_{20}$ to $\mathbb{Z}_{30}$. (6)

(c) Let n be an integer with decimal representation $a_k a_{k-1} \cdots a_1 a_0$. Prove that n is divisible by 11 if and only if $a_0 - a_1 + a_2 - \cdots + (-1)^k a_k$ is divisible by 11. (6)

(d) Show that a homomorphism from a field onto a ring with more than one element must be an isomorphism. (6)

5. (a) Let W_1 and W_2 be subspaces of a vector space V. Prove that $W_1 + W_2$ is a smallest subspace of V that contains both W_1 and W_2. (6)

(b) For the following polynomials in $P_3(\mathbb{R})$, determine whether the first polynomial can be expressed as linear combination of other two. (6)

$$\{x^3 - 8x^2 + 4x,\ x^3 - 2x^2 + 3x - 1,\ x^3 - 2x + 3\}.$$

(c) Let $S = \{u_1, u_2, \ldots, u_n\}$ be a finite set of vectors. Prove that S is linearly dependent if and only if $u_1 = 0$ or $u_{k+1} \in \text{span}(\{u_1, u_2, \ldots, u_k\})$ for some $k\ (1 \le k < n)$. (6)

(d) Let $W_1 = \{(a, b, 0) \in \mathbb{R}^3 : a, b \in \mathbb{R}\}$ and $W_2 = \{(0, b, c)\ E\ \mathbb{R}^3 : b, c \in \mathbb{R}\}$ be subspaces of $\mathbb{R}^3$. Determine $\dim(W_1)$, $\dim(W_2)$, $\dim(W_1 \cap W_2)$ and $\dim(W_1 + W_2)$. Hence deduce that $W_1 + W_2 = \mathbb{R}^3$. Is $\mathbb{R}^3 = W_1 \oplus W_2$? (6)

6. (a) Let V and W be finite-dimensional vector spaces having ordered bases β and γ respectively, and let $T : V \to W$ be linear. Then for each $u \in V$, show

$$[T(u)]_\gamma = [T]_\beta^\gamma [u]_\beta. \quad (6½)$$

(b) Let $T: P_2(\mathbb{R}) \to P_3(\mathbb{R})$ be linear transformation defined by

$$T(a + bx + cx^2) = (a - c) + (a - c)x + (b - a)x^2 + (c - b)x^3.$$

Find null space N(T) and range space R(T). Also verify Rank-Nullity Theorem. (6½)

(c) For the matrix $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 5 & -4 \\ 1 & -2 & 2 \end{bmatrix}$ and ordered basis $\beta = \{(1, 1, 0), (0, 1, 1), (1, 2, 2)\}$, find $[L_A]_\beta$. Also find an invertible matrix Q such that $[L_A]_\beta = Q^{-1}AQ$. (6½)

(d) Let V and W be vector spaces and let T: V → W be linear. Prove that T is one-to-one if and only if T carries linearly independent subsets of V onto linearly independent subsets of W. (6½)

[This question paper contains 6 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1114 **A**

Unique Paper Code : 32351601

Name of the Paper : BMATH 613 – Complex Analysis

Name of the Course : **B.Sc. (H) Mathematics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. 3. **All** questions are compulsory.
3. Attempt **two** parts from each question.

1. (a) Sketch the region onto which the sector $r \leq 1$, $0 \leq \theta \leq \pi$ is mapped by the transformation $w = z^2$ and $w = z^3$. (6)

   (b) (i) Find the limit of the function $f(z) = \frac{(\bar{z})^2}{z}$ as z tends to 0.

P.T.O.

(ii) Show that $\lim_{z \to 1+\sqrt{3}i} \frac{z^2-2z+4}{z-1-\sqrt{3}i} = 2\sqrt{3}i.$ (3+3=6)

(c) Let u and v denote the real and imaginary components of the function f defined by means of the equations

$$f(z) = \begin{cases} \bar{z}^2/z & when\ z \neq 0 \\ 0 & when\ z = 0 \end{cases}$$

Verify that the Cauchy-Riemann equations are satisfied at the origin z = (0,0). (6)

(d) If $\lim_{z \to z_0} f(z) = F$ and $\lim_{z \to z_0} g(z) = G$, prove that $\lim_{z \to z_0} \frac{f(z)}{g(z)} = \frac{F}{G}$ if $G \neq 0$. (6)

2. (a) Find the values of z such that

(i) $e^z = 1 + \sqrt{3}i$, (ii) $e^{(2z-1)} = 1$. (3.5+3=6.5)

(b) Show that the roots of the equation $\cos z = 2$ are $z = 2n\pi + i\cosh^{-1}2$ $(n = 0, \pm1, \pm2, \ldots)$. Then express them in the form $z = 2n\pi \pm i\ln(2+\sqrt{3})$ $(n = 0, \pm1, \pm2, \ldots)$. (3.5+3=6.5)

(c) Show that (3.5+3=6.5)

(i) $\log(1+i)^2 = 2Log(1+i)$,

(ii) $\log(-1+\sqrt{3}i) = ln2 + 2\left(n+\frac{1}{3}\right)\pi i$ $(n = 0, \pm1, \pm2, \ldots)$

(d) Show that $\overline{\exp(iz)} = \exp(i\bar{z})$ if and only if $z = n\pi$ $(n = 0, \pm1, \pm2, \ldots)$. (6.5)

3. (a) (i) State mean value theorem of integrals.

Does it hold true for complex valued functions? Justify.

(ii) Evaluate $\int_0^{2\pi} e^{im\theta} e^{-in\theta} d\theta$. (3+3=6)

(b) Parametrize the curves C_1 and C_2, where

C_1: Semicircular path from −1 to 1

C_2: Polygonal path from the vertices −1, −1+i, 1+i and 1

Evaluate $\int_{C_1} z\, dz$ and $\int_{C_2} z\, dz$. (3+3=6)

(c) For an arbitrary smooth curve C: z = z(t), a ≤ t ≤ b, from a fixed point z_1 to another fixed point z_2, show that the value of the integrals

(i) $\int_{z_1}^{z_2} z\, dz$ and

(ii) $\int_{z_1}^{z_2} dz$

depend only on the end points of C. (3+3=6)

(d) State ML inequality theorem. Use it to prove that $\left|\int_C \frac{dz}{z^4}\right| \leq 4\sqrt{2}$, where C denotes the line segment from z = i to z = 1. (2+4=6)

4. (a) A function f(z) is continuous on a domain D such that all the integrals of f(z) around closed contours lying entirely in D have the value zero. Prove that f(z) has an antiderivative throughout D. (6.5)

(b) State Cauchy Goursat theorem. Use it to evaluate the integrals

(i) $\int_C \frac{1}{z^2+2z+2}dz$, where C is the unit circle $|z| = 1$

(ii) $\int_C \frac{2z}{z^2+2}dz$, where C is the circle $|z| = 2$

(2.5+2+2=6.5)

(c) State and prove Cauchy Integral Formula.

(2+4.5=6.5)

(d) (i) State Liouville's theorem. Is the function f(z) = cos z bounded? Justify.

(ii) Is it true that 'If p(z) is a polynomial in z then the function f(z) = 1/p(z) can never be an entire function'? Justify (4.5+2=6.5)

5. (a) If a series $\sum_{n=1}^{\infty} z_n$ of complex numbers converges then prove $\lim_{n\to\infty} z_n = 0$. Is the converse true? Justify. (6.5)

(b) Find the integral of $\int_C \frac{\cosh \pi z}{z^3+z}$ where C is the positively oriented circle $|z| = 2$. (6.5)

(c) Find the Taylor series representation for the function $f(z) = \frac{1}{z}$ about the point $z_0 = 2$. Hence prove that $\frac{1}{z^2} = \frac{1}{4}\sum_{n=0}^{\infty}(-1)^n(n+1)\left(\frac{z-2}{2}\right)^n$ for $|z-2| < 2$. (6.5)

(d) If a series $\sum_{n=0}^{\infty} a_n(z-z_0)^n$ converges to f(z) at all points interior to some circle $|z-z_0| = R$, then prove that it is the Taylor series for the function $f(z)$ in power of $z - z_0$. (6.5)

6. (a) For the given function $f(z) = \frac{z+1}{z^2+9}$ find the poles, order of poles and their corresponding residue. (6)

(b) Write the two Laurent Series in powers of z that represent the function $f(z) = \frac{1}{z+z^3}$ in certain domains and specify those domains. (6)

(c) Suppose that $z_n = x_n + iy_n$, $(n = 1,2,3,...)$ and $S = X + iY$. Then prove that $\sum_{n=1}^{\infty} z_n = S$ iff $\sum_{n=1}^{\infty} x_n = X$ and $\sum_{n=1}^{\infty} y_n = Y$. (6)

(d) Define residue at infinity for a function $f(z)$. If a function $f(z)$ is analytic everywhere in the finite plane except for a finite number of singular points interior to a positively oriented simple closed contour C, then prove that

$$\operatorname*{Res}_{z=\infty} f(z) = -\operatorname*{Res}_{z=0}\left[\frac{1}{z^2} f\left(\frac{1}{z}\right)\right].$$ (6)

[This question paper contains 10 printed pages.]

Your Roll No...............

Sr. No. of Question Paper :	**1211**	**A**
Unique Paper Code	: 32357614	
Name of the Paper	: DSE-3 MATHEMATICAL FINANCE	
Name of the Course	: **B.Sc. (Hons) Mathematics CBCS (LOCF)**	
Semester	: VI	

Duration : 3 Hours   Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory and carry equal marks.
4. Use of Scientific calculator, Basic calculator and Normal distribution tables all are allowed.

1. (a) Explain Duration of a zero-coupon bond. A 4-year bond with a yield of 10% (continuously compounded) pays a 9% coupon at the end of each year.

(i) What is the bond's price?

(ii) Use duration to calculate the effect on the bond's price of a 0.3% decrease in its yield?

(You can use the exponential values: $e^x = 0.9048$, 0.8187, 0.7408, and 0.6703 for x = −0.1, −0.2, −0.3, and −0.4, respectively)

(b) Explain Continuous Compounding. Suppose R_c denotes rate of interest with continuous compounding and R_m denotes equivalent rate with compounding m times per annum. Find the relation between R_c and R_m.

(c) An investor receives ₹ 1100 in one year in return for an investment of ₹ 1000 now. Calculate the percentage return per annum with :

(i) Annual compounding

(ii) Semi-annual compounding

(iii) Continuous compounding.

(You can use: ln(1.1) = 0.953)

(d) Define Bond Yield and Par Yield. Suppose that the 6-month, 12-month, 18-month, and 24-month zero rates are 5%, 6%, 6.5% and 7% respectively. What is the 2-year par yield? (You can use the exponential values: e^x = 0.9753, 0.9418, 0.9071, 0.8694 for x = −0.025, −0.06, −0.0975, −0.14, respectively.)

2. (a) Explain Hedging. A United States company expects to pay 1 million Canadian dollars in 6 months. Explain how the exchange rate risk can be hedged using

(i) A Forward Contract

(ii) An Option.

(b) (i) What is the difference between the over-the counter market and the exchange-traded market?

(ii) An investor enters a short forward contract to sell 175,000 British pounds for US dollars at an exchange rate of 1.900 US dollars per pound. How much does the investor gain or lose if the exchange rate at the end of the contract is 2.420?

(c) A 1-year forward contract on a non-dividend paying stock is entered into when the stock price is ₹ 40, and the risk-free rate of interest is 10% per annum with continuous compounding. What is the forward price? Justify using no arbitrage arguments. ($e^{0.1}$ =1.1052)

(d) (i) A trader writes an October call option with a strike price of ₹ 35. The price of the option is ₹ 6. Under what circumstances does the trader make a gain,

(ii) Suppose that you own 6,000 shares worth ₹ 75 each. How can put options be used to provide an insurance against a decline in the value of the holding over the next 4 months?

3. (a) Draw the diagrams illustrating the effect of changes in stock price, strike price, and expiration date on European call and put option prices when

S_0 = 50, K = 50, r = 5%, σ = 30%, and T = 1.

(b) Derive the put-call parity for European options on a non-dividend-paying stock. Use put-call parity to derive the relationship between the delta of a European call and the delta of a European put on a non-dividend-paying stock.

(c) An investor sells a European call on a share for ₹ 4. The stock price is ₹ 47 and the strike price is ₹ 50. Under what circumstances does the investor make a profit? Under what circumstances will the option be exercised? Draw a diagram showing the variation of the investor's profit with the stock price at the maturity of the option.

(d) Define upper bound and lower bound for European options on a non-dividend- paying stock. What is a lower bound for the price of a 3-month European put option on a non-dividend-paying stock when the stock price is ₹38, the strike price is ₹40, and the risk-free interest rate is 10% per annum? Justify using no arbitrage arguments. ($_e$-0.04 = 0.9753)

4. (a) A 4-month European call option on a dividend-paying stock is currently selling for ₹ 50. The stock

price is ₹ 640, the strike price is ₹ 600, and a dividend of ₹ 8 is expected in 1 month. The risk-free interest rate is 12% per annum for all maturities. What opportunities are there for an arbitrageur? ($e^{-0.04} = 0.9608$)

(b) Consider a one-period binomial model where the stock can either go up from S_0 to S_0u ($u > 1$) or down from S_0 to S_0d ($d < 1$). Suppose we have an option with payoff f_u if the stock moves up and payoff f_d if the stock moves down. By considering a portfolio consisting of long position in Δ shares of stock and a short position in the option, find the price of the option. Explain how the price can be expressed as an expected payoff discounted by the risk-free interest rate.

(c) A stock price is currently ₹ 50. It is known that at the end of two months it will be either ₹ 53 or ₹ 48. The risk-free interest rate is 12% per annum with continuous compounding. What is the value of a two-month European call option with a strike price of ₹ 49? Use no-arbitrage arguments. ($e^{0.02} = 1.0202$)

(d) Consider a two-period binomial model with current stock price S_0 = ₹ 100, the up factor $u = 1.3$, the down factor $d = 0.8$, $T = 1$ year and each period being of length six months. The risk-free interest rate is 5% per annum with continuous compounding. Construct the two-period binomial tree for the stock. Find the price of an American put option with strike K = ₹ 95 and maturity T = 1 year. ($e^{-0.025} = 0.9753$)

5. (a) Stock price in the Black-Scholes model satisfies

$$\ln S_T \sim \phi\left[\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma^2 T\right]$$

where $\phi(m, v)$ denotes a normal distribution with mean m and variance v. Find $Var[S_T]$.

(b) What is the price of a European put option on a non-dividend-paying stock when the stock price is ₹ 69, the strike price is ₹ 70, the risk-free interest rate is 5% per annum, the volatility is 35% per annum, and the time to maturity is six months?

(You can use exponential values: $e^{-0.0144} = 0.9857$, $e^{-0.025} = 0.9753$)

(c) Let V be a lognormal random variable with ω being the standard deviation of ln V. Prove that

$$E[\max(V - K, 0)] = E(V)N(d_1) - KN(d_2)$$

where

$$d_1 = \frac{\ln\left[\frac{E(V)}{K}\right] + \frac{\omega^2}{2}}{\omega}, \quad d_2 = \frac{\ln\left[\frac{E(V)}{K}\right] - \frac{\omega^2}{2}}{\omega}$$

and E denotes the expected value. Use this result to derive the Black-Scholes formula for the price of a European call option on a non-dividend paying stock.

(d) A stock price is currently ₹ 50. Assume that the expected return from the stock is 18% and its volatility is 30%. What is the probability distribution for the stock price in 2 years? Calculate the mean and standard deviation of the distribution. ($e^{0.18} = 1.1972$)

6. (a) Discuss gamma of a portfolio of options and calculate the gamma of a European call option on a non-dividend-paying stock where the stock price is ₹ 49, the strike price is ₹ 50, the risk-free

interest rate is 5% per annum and the time to maturity is 20 weeks, and the stock price volatility is 30% per annum. ($\ln(49/50) = -0.0202$)

(b) What is the relationship between delta, theta and gamma of an option? Show by substituting for various terms in this relationship that it is true for a single European put option on a non-dividend-paying stock.

(c) Find the payoff from a bear spread created using put options. Also draw the profit diagram corresponding to this trading strategy.

(d) Companies X wishes to borrow US dollars at a fixed interest rate. Company Y wishes to borrow Indians rupees at a fixed rate of interest. The amounts required by the two companies are roughly the same at the current exchange rate. The companies have been quoted the following interest rates, which have been adjusted for the impact of taxes :

	Rupees	Dollars
Company X	9.6%	6.0%
Company Y	11.1%	6.4%

Design a swap that will net a bank, acting as intermediary, 50 basis points per annum. Make the swap equally attractive to the two companies and ensure that all foreign exchange risk is assumed by the bank.

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1299 **A**

Unique Paper Code : 32357616

Name of the Paper : DSE-4 Linear Programming and Applications

Name of the Course : **CBCS (LOCF) – B.Sc. (H) Mathematics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions carry equal marks.

1. (a) Solve the following Linear Programming Problem by Graphical Method :

P.T.O.

Minimize $3x + 2y$

subject to $5x + y \geq 10$

$x + y \geq 6$

$x + 4y \geq 12$

$x \geq 0, y \geq 0$.

(b) Define a Convex Set. Show that the set S defined as :

$$S = \{(x,y) \mid x^2 + y^2 \leq 4\} \text{ is a Convex Set.}$$

(c) Find all basic feasible solutions of the equations:

$$x_1 + x_2 + 2x_3 + 3x_4 = 12$$

$$x_2 + 2x_3 + x_4 = 8$$

(d) Prove that to every basic feasible solution of the Linear Programming Problem:

Minimize $z = cx$

subject to $Ax = b, x \geq 0$

there corresponds an extreme point of the feasible region.

2. (a) Let us consider the following Linear Programming Problem:

Minimize $z = cx$

subject to $Ax = b, x \geq 0$

Let $(x_B, 0)$ be a basic feasible solution corresponding to a basis B having an a_j with $z_j - c_j > 0$ and all corresponding enteries $y_{ij} \leq 0$, then show that Linear Programming Problem has an unbounded solution.

(b) Let $x_1 = 2, x_2 = 1, x_3 = 1$ be a feasible solution to the system of equations:

$$x_1 + 4x_2 - x_3 = 5$$

$$2x_1 + 3x_2 + x_3 = 18$$

Is this a basic feasible solution? If not, reduce it to two different basic feasible solutions.

(c) Using Simplex method, find the solution of the following Linear Programming Problem:

Minimize $x_1 - 3x_2 + 2x_3$

subject to $3x_1 - x_2 + 2x_3 \leq 7$

$2x_1 - 4x_2 \geq -12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

$x_1, x_2, x_3 \geq 0$.

(d) Solve the following Linear Programming Problem by Big-M method:

Maximize $x_1 - 4x_2 + 3x_3$

subject to

$2x_1 - x_2 + 5x_3 = 40$

$x_1 + 2x_2 - 3x_3 \geq 22$

$3x_1 + x_2 + 2x_3 = 30$

$x_1, x_2, x_3 \geq 0.$

3. (a) Solve the following Linear Programming Problem by Two Phase Method :

Maximize $x_1 + 4x_2 + 3x_3$

subject to

$x_1 + x_2 + x_3 \geq 4$

$-2x_1 + 3x_2 - x_3 \leq 2$

$x_2 - 2x_3 \leq 1$

$x_1, x_2, x_3 \geq 0.$

(b) Find the solution of given system of equations using Simplex Method:

$3x_1 - 2x_2 = 8$

$x_1 + 2x_2 = 4$

Also find the inverse of A where $A = \begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix}$.

(c) Using Simplex method, find the solution of the following Linear Programming Problem :

Maximize $2x_1 + x_2$

subject to

$x_1 - x_2 \leq 10$

$2x_1 - x_2 \leq 40$

$x_1 \geq 0,\ x_2 \geq 0.$

(d) Find the optimal solution of the Assignment Problem with the following cost matrix :

Job \ Machines	I	II	III	IV	V	VI
A	4	8	5	4	6	9
B	8	3	8	4	11	7
C	9	5	7	9	8	7
D	10	9	5	6	9	9
E	3	11	9	10	10	9
F	9	5	7	10	8	7

4. (a) Find the Dual of following Linear Programming Problem :

Minimize $x_1 + x_2 + 3x_3$

subject to

$4x_1 + 8x_2 \geq 3$

$7x_2 + 4x_3 \leq 6$

$3x_1 - 2x_2 + 5x_3 = 7$

$x_1 \leq 0,\ x_2 \geq 0,\ x_3$ is unrestricted.

(b) State and prove the Weak Duality Theorem. Also show that if the objective function values corresponding to feasible solutions of the Primal and Dual Problem are equal then the respective solutions are optimal for the respective Problems.

(c) Using Complementary Slackness Theorem, find optimal solutions of the following Linear Programming Problem and its Dual:

Maximize $4x_1 + 3x_2$
subject to

$$x_1 + 2x_2 \le 2$$
$$x_1 - 2x_2 \le 3$$
$$2x_1 + 3x_2 \le 5$$
$$x_1 + x_2 \le 2$$
$$3x_1 + x_2 \le 3$$
$$x_1, x_2 \ge 0.$$

(d) For the following cost minimization Transportation Problem find initial basic feasible solutions by using North West Comer rule, Least Cost Method and Vogel's Approximation Method. Compare the three solutions (in terms of the cost):

Source \ Destination	A	B	C	D	E	Supply
I	15	15	16	17	15	24
II	18	19	16	20	15	38
III	16	15	22	17	20	43
Demand	27	12	32	17	17	

5. (a) Solve the following cost minimization Transportation Problem :

Origin \ Destinations	I	II	III	IV	Availability
A	10	11	10	13	30
B	12	12	11	10	50
C	13	11	14	18	20
Requirements	20	40	30	10	

(b) Four new machines are to be installed in a machine shop and there are five vacant places available. Each machine can be installed at to one and only one place. The cost of installation of each job on each place is given in table below. Find the Optimal Assignment. Also find which place remains vacant.

Machine \ Place	A	B	C	D	E
I	13	15	19	14	15
II	16	13	13	14	13
III	14	15	18	15	11
IV	18	12	16	12	10

(c) Define Maxmin and Minmax value for a Fair Game. Using Maxmin and Minmax Principle, find the saddle point, if exists, for the following pay – off matrix :

$$\text{Player 2}\overset{\text{Player 1}}{\begin{bmatrix} 1 & 3 & 6 \\ 5 & 4 & 2 \\ 2 & 1 & 3 \end{bmatrix}}.$$

(d) Convert the following Game Problem into a Linear Programming Problem for player A and player B and solve it by Simplex Method :

$$\text{Player A}\overset{\text{Player B}}{\begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 6 \end{bmatrix}}.$$

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1298 **A**

Unique Paper Code : 32357610

Name of the Paper : DSE-4 (Number Theory)

Name of the Course : **CBCS (LOCF) – B.Sc. (H) (Mathematics)**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts of each question.
4. Question Nos. **1** to **3**, each part carries **6.5** marks and Question Nos. **4** to **6**, each part carries **6** marks.

1. (a) Determine all solutions in the integers of the Diophantine equation $24x + 138y = 18$.

   (b) A farmer purchased 100 head of livestock for a total cost of Rs. 4000. Prices were as follow: calves, Rs. 120 each; lambs, Rs. 50 each; piglets, Rs. 25 each. If the farmer obtained at least one animal of each type, how many of each did he buy?

(c) Write a short note on Goldbach conjecture.

(d) Find the remainder obtained upon dividing the sum $1! + 2! + 3! + \cdots + 100!$ by 12.

2. (a) Prove that the congruences

$x \equiv a \pmod{n}$ and $x \equiv b \pmod{m}$ admits a simultaneous solution if $\gcd(n,m) | (a-b)$; if a solution exists, confirm that it is unique modulo lcm(n,m).

(b) Solve the linear congruence $25x \equiv 15 \pmod{29}$.

(c) If p an q are distinct primes, prove that

$$p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}.$$

(d) State and prove Wilson's theorem.

3. (a) If f is a multiplicative function and F is defined by $F(n) = \sum_{d/n} f(d)$ then show that F is also multiplicative, explain your result when m = 8 and n = 3.

(b) Explain Mobius p- function with example and also show that

$$\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0$$

for each positive integer n.

(c) Use the fact that each prime p has a primitive root to give a different proof of Wilson's theorem.

(d) Let r be a primitive root of the integer n. Prove that r^k is a primitive root of n if and only if $\gcd(k, \Phi(n)) = 1$.

4. (a) Define number – theoretic function and also show that number – theoretic functions σ and τ both are multiplicative functions.

(b) Write a short note on Mobius function and show this function is multiplicative function.

(c) If p is a prime and k > 0, then prove $\phi(p^k) = p^k - p^{k-1}$. Explain your result by an example.

(d) Use Euler's theorem for any odd integer a, to prove $a^{33} \equiv a \pmod{4080}$.

5. (a) Find a primitive root for any integer of the form 17^k.

(b) Let p be an odd prime and gcd(a, p) = 1. Then prove that 'a' is a quadratic residue of p if and only if $a^{(p-1)/2} \equiv 1 \pmod{p}$.

(c) Let p be an odd prime and let a and b be integers that are relatively prime to p. Show that (ab/p) = (a/p) (b/p).

(d) Find the value of Legendre symbols (18/43) and (−72/131).

6. (a) Use Gauss lemma to compute Legendre symbol (5/19).

(b) Show that 7 and 18 are the only incongruent solutions of $x^2 \equiv -1 \pmod{5^2}$.

(c) Using the linear cipher $C \equiv 5P + 11 \pmod{26}$ encrypt the message CRYPTOGRAPHY.

(d) Use the Hill's cipher

$$C_1 \equiv 5P_1 + 2P_2 \pmod{26}$$

$$C_2 \equiv 3P_1 + 4P_2 \pmod{26}$$

to encrypt the message GIVE THEM TIME.

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1608 **A**

Unique Paper Code : 42357602

Name of the Paper : DSE – Probability and Statistics

Name of the Course : **B.Sc. Mathematical Sciences**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt all the **six** questions.
3. Each question has **four** parts. Attempt any **two** parts from each question.
4. Each part in Question **1, 3, 5** carries **6** marks.
5. Each part in Question **2, 4, 6** carries **6.5** marks.
6. Use of scientific calculator is allowed.

1\. (a) A secretary types three letters and three corresponding envelopes. In a hurry, he places at random one letter in each envelope. What is the probability that at least one letter is in correct envelope?

P.T.O.

(b) Let $\{C_n\}$ be a nondecreasing sequence of events. Show that

$$\lim_{n\to\infty} P(C_n) = P\left(\lim_{n\to\infty} C_n\right) = P\left(\bigcup_{n=1}^{\infty} C_n\right).$$

(c) Let $p_X(x)$ be the pmf of a random variable X. Find and sketch the cdf $F_X(x)$ of X, where

$$p_X(x) = \begin{cases} \frac{x}{15} & , x = 1, 2, 3, 4, 5 \\ 0 & \text{elsewhere} \end{cases}$$

Also find

(i) $P(X = 1 \text{ or } 2)$

(ii) $P\left(\frac{1}{2} < X < \frac{5}{2}\right)$

(iii) $P(1 \le X \le 4)$

(d) Define cumulative distribution function. Let X be a random variable with cumulative distribution function F(x). Show that

$$\lim_{x \downarrow x_0} F(x) = F(x_0)$$

for all $x_0 \in R$.

2. (a) A bowl contains 10 chips, of which 8 are marked \$2 each and 2 are marked \$5 each. A person chooses three chips at random without replacement from this bowl. If person is to receive the sum of the resulting amounts. Find his expectation.

(b) Find the moment generating function of Normal Distribution. Also, find its mean and variance using moment generating function.

(c) Let a random variable of continuous type have a pdf f(x) whose graph is symmetric with respect to the line x = c. If the mean value of X exists. Show that E(X) = c.

(d) If the probability is 0.75 that a person will believe a rumor about a certain actor. Find the probability that

(i) the fifth person to hear the rumor will be the second to believe it.

(ii) the sixth person to hear the rumer will be the fourth to believe it.

3. (a) Find the joint probability density of the two random variables X and Y whose joint distribution function is given by :

$$F_{X,Y}(x,y)=\begin{cases}\left(1-e^{-x^2}\right)\left(1-e^{-y^2}\right); & x>0,\ y>0\\ 0 & :\ \text{otherwise}\end{cases}$$

Then use the joint pdf to obtain $P[1 < X \le 2, 1 < Y \le 2]$.

(b) Let X and Y have the joint probability mass function :

$$p(m,n)=\begin{cases}\dfrac{1}{2^{(m+1)}}; & m \ge n\\ 0 & :\ m < n\end{cases}$$

for m, n = 1,2,.... Verify that it satisfies the properties of a joint pmf. Compute the marginal probability mass functions.

(c) Consider the experiment of tossing two tetrahedra with sides numbered 1 to 4. Let X denotes the smaller of the two downturned numbers and Y be the larger.

(i) Find the joint mass function of X and Y.

(ii) Find P [$X \ge 2, Y \ge 2$].

(iii) Find conditional pmf of X given Y.

(d) Suppose that X and Y are jointly continuous random variables with joint density :

$$f_{X,Y(x,y)}=\begin{cases}c\,x^2y: & 0<x<y<2\\ 0\ : & \text{otherwise}\end{cases}$$

(i) Find the value of c ?

(ii) What is the probability that $X < 2Y$?

(iii) What are the marginal densities f_X and f_Y ?

4. (a) Let X and Y be two random variables with joint pdf :

$$f(x,y)=\begin{cases}5\,x\,y: & 0<x,\ y<1\\ 0\ : & \text{otherwise}\end{cases}$$

(i) Find joint moment generating function of X and Y.

(ii) Using joint mgf, compute E(XY) and E(X).

(iii) Compute E(2X – 4XY).

(b) If the joint probability density of X and Y is given by :

$$f(x,y)=\begin{cases} 24y(1-x-y) & : \quad x>0,\ y>0,\ x+y<1 \\ 0 & \quad \text{otherwise} \end{cases}$$

Compute conditional mean and variance of Y given $X=x,\ x>0$.

(c) Let X and Y be discrete random variables. Then prove the following :

(i) $E[E(Y \mid X)] = E(Y)$

(ii) $var(Y) = E[var(Y \mid X)] + var(\ E(Y \mid X))$.

(d) If two cards are randomly drawn (without replacement) from an ordinary deck of 52 playing cards, X is the number of spades obtained in the first draw, and Y is the total number of spades obtained in both draws, find

(i) the joint cumulative distribution function of X and Y;

(ii) conditional cdf of X given $Y = y$.

5. (a) If X is a random variable with mean μ and variance σ^2, then prove that for any $k > 0$

$$P[|X-\mu|<k\sigma] \geq 1-\frac{1}{k^2}$$

(b) Use the Central limit theorem to prove that if X is a random variable having binomial distribution with parameters n and θ, then

$$\frac{X-n\theta}{\sqrt{n\theta(1-\theta)}} \sim N(0,1) \text{ as } n \to \infty.$$

(c) Given the joint density

$$f(x,y)=\begin{cases} 2 & \text{for } 0<y<x<1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the regression equation of Y on X.

(d) Using method of least squares fit a straight line for the following data

X	1	2	3	4	6	8
Y	2.4	3	3.6	4	5	6

6. (a) Let X and Y have joint probability mass function described as follows

(x, y)	(1, 1)	(1, 2)	(1, 3)	(2, 1)	(2, 2)	(2, 3)
p(x, y)	$\frac{2}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{4}{15}$

Find the coefficient of correlation of X and Y.

(b) Find $P\left[0 < X < \frac{1}{3}, 0 < Y < \frac{1}{3}\right]$, if the random variable X and Y have joint probability density function

$$f(x, y) = \begin{cases} 4x(1-y) & 0 < x < 1,\ 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

(c) If the regression of Y on X is linear, then show that

$$E[Y|X] = \mu_2 + \frac{\rho\,\sigma_2}{\sigma_1}(X - \mu_1).$$

(d) Let X_i, i = 1 to 5 be independent random variables, each being uniformly distributed over (0,1). Use the Markov's inequality to get bound on

$$P[X_1 + X_2 + X_3 + X_4 + X_5 \geq 8].$$

[This question paper contains 6 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1359 **A**

Unique Paper Code : 32351602

Name of the Paper : BMATH614: Ring Theory and Linear Algebra II

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) (i) If D is an Integral domain, prove that D[x] is an integral domain.

(ii) If R is a commutative ring, prove that the characteristic of R[x] is same as the characteristic of R.

(b) Let $f(x) = 5x^4 + 3x^3 + 1$ and $g(x) = 3x^2 + 2x + 1$ in $Z_7[x]$. Compute the product f(x)g(x). Determine the quotient and the remainder upon dividing f(x) by g(x).

P.T.O.

(c) Let F be a field and let $I = \{a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0 | a_i \in F$ and $f(1) = a_n + \ldots a_0 = 0\}$. Prove that I is an Ideal of F[x] and find a generator of I.

(d) Let R[x] denote the ring of polynomials with real coefficients. Then prove that $\frac{R[x]}{\langle x^2+1\rangle}$ is isomorphic to the ring of complex numbers.

(3+3.5,6.5,6.5,6.5)

2. (a) (i) Let F be a field and $p(x) \in F[x]$ be irreducible over F. Prove that $\langle p(x) \rangle$ is a maximal ideal in F[x].

(ii) Show that, $\frac{Z_2[x]}{\langle x^3+x+1\rangle}$ is a field with 8 elements.

(b) Determine which of the polynomials below are irreducible over Q.

(i) $3x^5 + 15x^4 - 20x^3 + 10x + 20$

(ii) $x^4 + x + 1$

(c) In integral domain $Z[\sqrt{-3}]$, prove that $1+\sqrt{-3}$ is irreducible but not prime.

(d) Define Euclidean domain. Prove that every Euclidean domain is a principal ideal domain.

(3+3,3+3,6,6)

3. (a) Let $V = P_1(R)$ and V^* denote the dual space of V. For $p(x) \in V$, define

$f_1, f_2 \in V^*$ by $f_1(p(x)) = \int_0^1 p(t)dt$ and $f_2(p(x)) = \int_0^2 p(t)dt$. Prove that $\{f_1, f_2\}$ is a basis for V^* and find a basis for V for which it is the dual basis.

(b) Let W be a subspace of finite dimensional vector space V. Prove that

$\dim(W) + \dim(W^0) = \dim(V)$, where W^0 is annihilator of W.

(c) Let T be a linear operator on $M_{n\times n}(R)$ defined by $T(A) = A^t$. Show that ±1 are the only eigenvalues of T. Find the eigenvectors corresponding to each eigenvalue. Also find bases for $M_{2\times 2}(R)$ consisting of eigenvectors of T.

(d) Let T be a linear operator on R^3 defined by T(a, b, c) = (3a + b, 3b + 4c, 4c). Show that T is digonalizable by finding a basis for R^3 consisting of eigen vectors of T. (6.5,6.5,6.5,6.5)

4. (a) Let T be a linear operator on finite dimensional vector space V and let W be the T-cyclic subspace of V generated by a non-zero vector $v \in V$. Let k = dim (W). Then prove that $\{v, T(v), \ldots\ldots, T^{k-1}(v)\}$ is basis for W.

(b) State Cayley Hamilton Theorem. Verify the theorem for linear operator T: $R^2 \rightarrow R^2$ defined by T(a, b) = (a + 2b, –2a + b).

(c) Let T be a linear operator on R^3 defined by T(a, b, c) = (3a – b, 2b, a – b + 2c). Find the characteristic polynomial and minimal polynomial of T.

(d) (i) Let T be an invertible linear operator. Prove that a scalar λ is an eigen value of T if and only if λ^{-1} is an eigenvalue of T^{-1}.

(ii) Prove that similar matrices have the same characteristic polynomial. (6,6,6,3+3)

5. (a) Show that in a complex inner product space V over field F. For x, y ∈ V, prove the following identities

(i) $\langle x, y\rangle = \frac{1}{4}\|x+y\|^2 - \frac{1}{4}\|x-y\|^2$ if F = R

(ii) $\langle x, y\rangle = \frac{1}{4}\sum_{k=1}^{4} i^k \left\|x + i^k y\right\|^2$ if F = C, where $i^2 = -1$.

(b) Let V be an inner product space, and let $S = \{v_1, v_2, \ldots, v_n\}$ be an orthonormal subset of V. Prove the Bessel's Inequality:

$$\|x\|^2 \geq \sum_{i=1}^{n} |\langle x, v_i\rangle|^2 \text{ for any } x \in V.$$

Further prove that Bessel's Inequality is an equality if and only if x ∈ span(S).

(c) Let V = $P_2(R)$, with the inner product

$$\langle f(x), g(x)\rangle = \int_0^1 f(t)g(t)\,dt$$

and with the standard basis $\{1, x, x^2\}$. Use Gram-Semidth process to obtain an orthonormal basis β of $P_2(R)$. Also, compute the Fourier coefficients of h(x) = 1 + x relative to β.

(d) Find the minimal solution to the following system of linear equations

$x + 2y - z = 1$

$2x + 3y + z = 2$

$4x + 7y - z = 4$ (3+3.5,6.5,6.5,6.5)

6. (a) For the data {(−3, 9), (−2, 6), (0, 2), (1, 1)}, use the least squares approximation to find the best fit with a linear function and compute the error E.

(b) Let T be a linear operator on a finite dimensional inner product space V. Suppose that the characteristic polynomial of T splits. Then prove that there exists an orthonormal basis β for V such that the matrix $[T]_\beta$ is upper triangular.

(c) (i) Let T be a linear operator on $\mathbb{C}^2$ defined by $T(a, b) = (2a + ib, a + 2b)$. Determine whether T is normal, self-adjoint, or neither.

(ii) For $z \in \mathbb{C}$, define $T_z: \mathbb{C} \to \mathbb{C}$ by $T_z(u) = zu$. Characterize those z for which T_z is normal, self adjoint, or unitary.

(d) Let U be a Unitary operator on an inner product space V and let W be a finite dimensional U-invariant subspace of V. Then, prove that

(i) $U(W) = W$

(ii) $W^\perp$ is U-invariant (6,6,3+3,3+3)

(3000)